OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4730

Mechanics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Tuesday 15 June 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

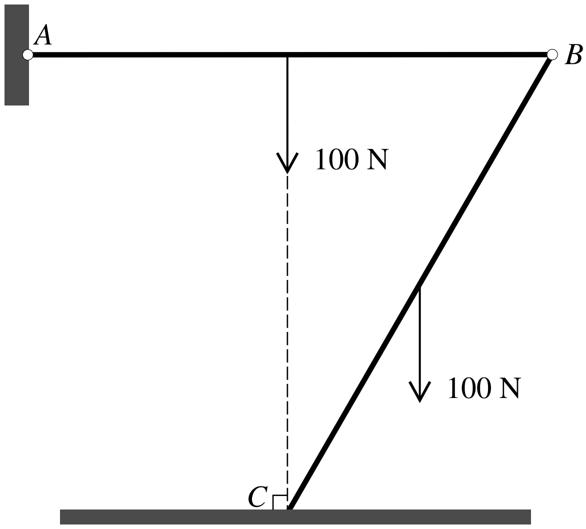
- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 A small ball of mass 0.8 kg is moving with speed $10.5\,\mathrm{m\,s^{-1}}$ when it receives an impulse of magnitude 4 N s. The speed of the ball immediately afterwards is $8.5\,\mathrm{m\,s^{-1}}$. The angle between the directions of motion before and after the impulse acts is α. Using an impulse-momentum triangle, or otherwise, find α. **[6]**

2



Two uniform rods *AB* and *BC* are of equal length and each has weight 100 N. The rods are freely jointed to each other at *B*, and *A* is freely jointed to a fixed point. The rods are in equilibrium in a vertical plane with *AB* horizontal and *C* resting on a rough horizontal surface. *C* is vertically below the mid-point of *AB* (see diagram).

(i) By taking moments about *A* for *AB*, find the vertical component of the force on *AB* at *B*. Hence find the vertical component of the contact force on *BC* at *C*. **[3]**

(ii) Calculate the magnitude of the frictional force on *BC* at *C* and state its direction. **[4]**

3

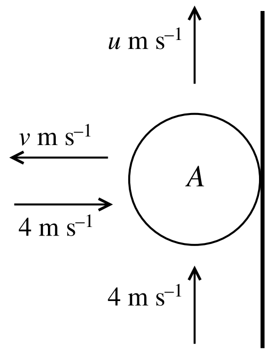


Fig. 1

A uniform smooth sphere A moves on a smooth horizontal surface towards a smooth vertical wall. Immediately before the sphere hits the wall it has components of velocity parallel and perpendicular to the wall each of magnitude $4\,\mathrm{m\,s^{-1}}$. Immediately after hitting the wall the components have magnitudes $u\,\mathrm{m\,s^{-1}}$ and $v\,\mathrm{m\,s^{-1}}$, respectively (see Fig. 1).

(i) Given that the coefficient of restitution between the sphere and the wall is $\frac{1}{2}$, state the values of u and v. **[2]**

Shortly after hitting the wall the sphere A comes into contact with another uniform smooth sphere B, which has the same mass and radius as A. The sphere B is stationary and at the instant of contact the line of centres of the spheres is parallel to the wall (see Fig. 2). The contact between the spheres is perfectly elastic.

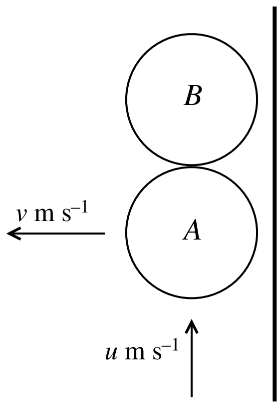


Fig. 2

(ii) Find, for each sphere, its speed and its direction of motion immediately after the contact. **[6]**

4 O is a fixed point on a horizontal plane. A particle P of mass 0.25 kg is released from rest at O and moves in a straight line on the plane. At time t s after release the only horizontal force acting on P has magnitude

$$\tfrac{1}{2400}(144 - t^2)\text{ N} \quad \text{for } 0 \leqslant t \leqslant 12$$

and

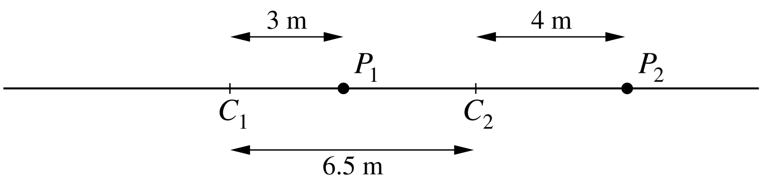
$$\tfrac{1}{2400}(t^2 - 144)\text{ N} \quad \text{for } t \geqslant 12.$$

The force acts in the direction of P's motion. P's velocity at time t s is $v\text{ m s}^{-1}$.

(i) Find an expression for v in terms of t, valid for $t \geqslant 12$, and hence show that v is three times greater when $t = 24$ than it is when $t = 12$. **[8]**

(ii) Sketch the (t, v) graph for $0 \leqslant t \leqslant 24$. **[3]**

5



Particles P_1 and P_2 are each moving with simple harmonic motion along the same straight line. P_1's motion has centre C_1, period 2π s and amplitude 3 m; P_2's motion has centre C_2, period $\frac{4}{3}\pi$ s and amplitude 4 m. The points C_1 and C_2 are 6.5 m apart. The displacements of P_1 and P_2 from their centres of oscillation at time t s are denoted by x_1 m and x_2 m respectively. The diagram shows the positions of the particles at time $t = 0$, when $x_1 = 3$ and $x_2 = 4$.

(i) State expressions for x_1 and x_2 in terms of t, which are valid until the particles collide. **[3]**

The particles collide when $t = 5.99$, correct to 3 significant figures.

(ii) Find the distance travelled by P_2 before the collision takes place. **[4]**

(iii) Find the velocities of P_1 and P_2 immediately before the collision, and state whether the particles are travelling in the same direction or in opposite directions. **[4]**

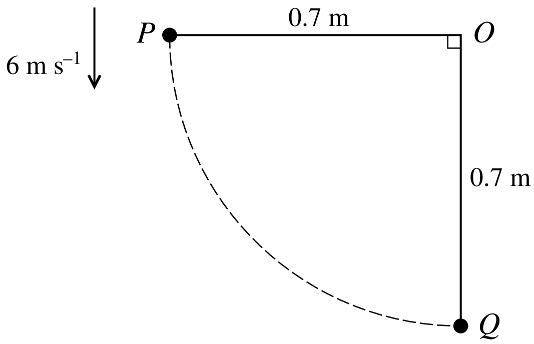
6 A bungee jumper of weight W N is joined to a fixed point O by a light elastic rope of natural length 20 m and modulus of elasticity 32 000 N. The jumper starts from rest at O and falls vertically. The jumper is modelled as a particle and air resistance is ignored.

(i) Given that the jumper just reaches a point 25 m below O, find the value of W. **[5]**

(ii) Find the maximum speed reached by the jumper. **[4]**

(iii) Find the maximum value of the deceleration of the jumper during the downward motion. **[3]**

7



A particle P is attached to a fixed point O by a light inextensible string of length 0.7 m. A particle Q is in equilibrium suspended from O by an identical string. With the string OP taut and horizontal, P is projected vertically downwards with speed $6\,\text{m}\,\text{s}^{-1}$ so that it strikes Q directly (see diagram). P is brought to rest by the collision and Q starts to move with speed $4.9\,\text{m}\,\text{s}^{-1}$.

(i) Find the speed of P immediately before the collision. Hence find the coefficient of restitution between P and Q. **[3]**

(ii) Given that the speed of Q is $v\,\text{m}\,\text{s}^{-1}$ when OQ makes an angle θ with the downward vertical, find an expression for v^2 in terms of θ, and show that the tension in the string OQ is $14.7m(1 + 2\cos\theta)$ N, where m kg is the mass of Q. **[6]**

(iii) Find the radial and transverse components of the acceleration of Q at the instant that the string OQ becomes slack. **[4]**

(iv) Show that $V^2 = 0.8575$, where $V\,\text{m}\,\text{s}^{-1}$ is the speed of Q when it reaches its greatest height (after the string OQ becomes slack). Hence find the greatest height reached by Q above its initial position. **[4]**

1	For included angle marked α or for $0.8(10.5 – 8.5\cos\alpha) = 4\cos\beta$ For opposite side marked 4/0.8 (or 4) or for -- $0.8 \times 8.5\sin\alpha = 4\sin\beta$ $8.4^2 + 6.8^2 – 2 \times 8.4 \times 6.8 \cos\alpha = 4^2$ $\alpha = 28.1^\circ$	M1 A1 A1 M1 A1ft A1 [6]	For triangle with two of its sides marked 0.8 x 10.5 and 0.8 x 8.5 (or 10.5 and 8.5) or for using I = Δmv in one direction. Allow B1 for omission of 0.8 Allow B1 for omission of 0.8 For using the cosine rule or for eliminating β ft 0.8 mis-used or not used
2(i)	$[100a = 2aV_B]$ Vertical component at B is 50 N Vertical component at C is 150 N	M1 A1 A1 [3]	For taking moments about A for AB
(ii)	$100(0.5a) + (\sqrt{3}\,a)F = 150a$ or $100a + 100(1.5a) = 150a + (\sqrt{3}\,a)F$ Frictional force is 57.7 N Direction is to the right	M1 A1ft A1 B1 [4]	For taking moments about B for BC (3 terms needed) or about A for the whole (4 terms needed)
3(i)	u = 4 v = 2	B1 B1 [2]	
(ii)	mu = ma + mb (or u = b – a) u = b – a (or mu = ma + mb) a = 0 and b = 4ms^{-1} Speed of A is 2ms^{-1} and direction at 90° to the wall Speed of B is 4ms^{-1} and direction parallel to the wall	M1 A1 B1 A1ft A1ft A1ft [6]	For using the principle of conservation of momentum or for using NEL with e = 1 ft incorrect u ft incorrect v ft incorrect u
4(i)	$[0.25\, dv/dt = 3/50 – t^2/2400]$ $v = 12t/50 – t^3/1800$ $[v(12) = 1.92]$ $[0.25\, dv/dt = t^2/2400 - 3/50 \rightarrow v = t^3/1800 - 12t/50 + C_2]$ $[1.92 = 0.96 – 2.88 + C_2]$ $v = t^3/1800 - 12t/50 + 3.84$ $v(24) = 5.76 = 3 \times v(12)$	M1 M1 A1 M1 M1 M1 A1 A1 [8]	For using Newton's second law (1st or 2nd stage) For attempting to integrate (1st stage) and using v(0) = 0 (may be implied by the absence of + C_1) For evaluating v when force is zero For using Newton's second law (2nd stage) and integrating For using v(12) = 1.92 AG

(ii)	Sketch has v(0) = 0 and slope decreasing (convex upwards) for $0 < t < 12$ Sketch has slope increasing (concave upwards) for $12 < t < 24$ Sketch has v(t) continuous, single valued and increasing (except possibly at t = 12) with v(24) seen to be > 2v(12)	B1 B1 B1 [3]	
5(i)	For using amplitude as a coefficient of a relevant trigonometric function. For using the value of ω as a coefficient of t in a relevant trigonometric function. $x_1 = 3\cos t$ and $x_2 = 4\cos 1.5t$	B1 B1 B1 [3]	
(ii)	Part distance is 20m [20 – (-3.62)] Distance travelled by P_2 is 23.6 m	M1 A1 M1 A1 [4]	For using distance travelled by P_2 for $0 < t < 5\pi/3$ is $5A_2$ For subtracting displacement of P_2 when t = 5.99 from part distance.
(iii)	$\dot{x}_1 = -3\sin t$; $\dot{x}_2 = -6\sin 1.5t$ $v_1 = 0.867$, $v_2 = -2.55$; opposite directions	M1 A1 M1 A1 [4]	For differentiating x_1 and x_2 For evaluating when t = 5.99 (must use radians)
	Alternative for (iii): $v_1^2 = 3^2 - 2.87^2$, $v_2^2 = 2.25[4^2 - (-3.62)^2]$ [$\pi < 5.99 < 2\pi \Rightarrow v_1 > 0$, $4\pi/3 < 5.99 < 2\pi \Rightarrow v_2 < 0$] $v_1 = 0.867$, $v_2 = -2.55$; opposite directions	M1 A1 M1 A1	For using $v^2 = n^2(a^2 - x^2)$ (must use radians to find values of x) For using the idea that v starts –ve and changes sign at intervals of T/2 s
6(i)	PE loss at lowest allowable point = 25W EE gain = $32000 \times 5^2/(2 \times 20)$ [25W = 20000] Value of W is 800	B1 M1 A1 M1 A1 [5]	For using EE = $\lambda x^2/(2L)$; may be scored in (i) or in (ii) For equating PE loss and EE gain and attempting to solve for W
(ii)	[800 = 32000x/20] $\frac{1}{2}(800/9.8)v^2 = 800 \times 20.5 - 32000 \times 0.5^2/(2 \times 20)$ Maximum speed is 19.9 ms^{-1}	M1 M1 A1 A1 [4]	For using W = $\lambda x/L$ at max speed For using the principle of conservation of energy (3 terms required)
(iii)	$(800)\ddot{x}/g = 800 - 32000 \times 5/20$ Max. deceleration is 88.2 ms^{-2}	M1 A1 A1 [3]	For applying Newton's second law to jumper at lowest point (3 terms needed)

7(i)	[½ mv^2 – ½ m 6^2 = mg(0.7)] Speed of P before collision is $7.05ms^{-1}$ Coefficient of restitution is 0.695	M1 A1 B1ft [3]	For using the principle of conservation of energy for P (3 terms needed) ft 4.9 ÷ speed of P before collision
(ii)	[½ mv^2 = ½ m 4.9^2 - mg0.7(1 - cos θ)] v^2 = 3.43(3 + 4 cos θ) T – mgcos θ = mv^2/0.7 [T – m9.8cos θ = m3.43(3 + 4 cos θ)/0.7] Tension is 14.7m(1 + 2cos θ)N	M1 A1 M1 A1 M1 A1 [6]	For using the principle of conservation of energy for Q Accept any correct form For using Newton's second law radially with $a_r = v^2/r$ For substituting for v^2 AG
(iii)	T = 0 ➔ θ = 120° Radial acceleration is (±)4.9 ms^{-1} **or** transverse acceleration is (±)8.49 ms^{-1} Radial acceleration is (±)4.9 ms^{-1} **and** transverse acceleration is (±)8.49 ms^{-1}	B1 M1 A1 B1 [4]	For using a_r = -gcos θ {or 3.43(3 + 4cos θ)/0.7} or a_t = –gsin θ
			SR for candidates with a sin/cos mix in the work for M1 A1 B1 immediately above. (max. 1/3) Radial acceleration is (±)8.49 ms^{-1} **and** transverse acceleration is (±)4.9 ms^{-1} B1
(iv)	[V^2 = 3.43{3 + 4(-0.5)}x0.5^2 or V^2 = (-gcos120° x 0.7) x $cos^2 60°$] V^2 = 0.8575 [mgH = ½ m(4.9^2 – 0.8575) or mg(H – 1.05) = ½ m(3.43 – 0.8575)] Greatest height is 1.18 m	M1 A1 M1 A1 [4]	For using V = v(120°) x cos60° AG For using the principle of conservation of energy